In an investigation, a small object was suspended from the lower end of a vertical steel spring which was fixed at its upper end, as shown in Figure 1. A horizontal marker pin P was attached to the lower end of the spring. The vertical position, $x$, of the pin was measured against the millimetre scale of a metre rule clamped vertically in a fixed position. The measurement was made three times without, then with, the small object suspended from the spring.

(a) The readings obtained are shown in Table 1.

<table>
<thead>
<tr>
<th></th>
<th>$x$/mm</th>
<th>mean $x$/mm</th>
<th>extension $e$/mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>without the object on the spring</td>
<td>2 2 2</td>
<td>2.0</td>
<td>0</td>
</tr>
<tr>
<td>with the object on the spring</td>
<td>71 72 73</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(ii) Copy and complete Table 1 by calculating the mean vertical position of P and the extension of the spring when the object was placed on it. (iii) The readings were taken to a precision of 0.5 mm using a millimetre ruler. Estimate the percentage ‘uncertainty’ in the extension. (2 marks)

(b) The time period, $T$, of small vertical oscillations of the object on the spring was also measured by timing 20 oscillations three times. The timing readings for 20 oscillations were 10.98 s, 11.11 s and 10.97 s.

(i) Calculate the time period $T$. (ii) Use the readings to estimate the percentage ‘uncertainty’ in $T$. (2 marks)

(c) (i) Give an expression for the extension $e$ of the spring in terms of the mass $m$ of the object and the spring constant $k$ of the spring. (ii) Hence show that $T = 2\pi \sqrt{\frac{e}{g}}$. (3 marks)

(d) The experiment was repeated with objects of different mass suspended from the spring. The measurements obtained are given in Table 2. Plot a suitable graph using the above measurements to confirm the equation and to determine $g$. (9 marks)

<table>
<thead>
<tr>
<th>object</th>
<th>$e$/mm</th>
<th>$T$/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>70</td>
<td>0.551</td>
</tr>
<tr>
<td>2</td>
<td>139</td>
<td>0.761</td>
</tr>
<tr>
<td>3</td>
<td>205</td>
<td>0.923</td>
</tr>
<tr>
<td>4</td>
<td>271</td>
<td>1.062</td>
</tr>
<tr>
<td>5</td>
<td>341</td>
<td>1.187</td>
</tr>
<tr>
<td>6</td>
<td>409</td>
<td>1.291</td>
</tr>
</tbody>
</table>

(e) Discuss the accuracy of your determination of $g$. (4 marks)
The tuning fork shown in Figure 2 is labelled 512 Hz and has the tip of each of its two prongs vibrating with simple harmonic motion of amplitude 0.85 mm.

![Figure 2]

(a) (i) Figure 2 shows the extreme positions of the prongs. How is the distance marked \(d\) related to the amplitude of the prongs?
(ii) Sketch a graph to show how the displacement of one tip of the tuning fork changes with time. Mark each axis with an appropriate scale. (4 marks)

(b) (i) Calculate the maximum speed of the tip of a prong.
(ii) Calculate the maximum acceleration of the tip of a prong. (4 marks)

A simple pendulum consists of a 25 g mass tied to the end of a light string 800 mm long. The mass is drawn to one side until it is 20 mm above its rest position, as shown in Figure 3. When released it swings with simple harmonic motion.

![Figure 3]

(a) Calculate the period of the pendulum. (2 marks)

(b) Show that the initial amplitude of the oscillations is approximately 0.18 m, and that the maximum speed of the mass during the first oscillation is about 0.63 m s\(^{-1}\). (4 marks)

(c) Calculate the magnitude of the tension in the string when the mass passes through the lowest point of the first swing. (2 marks)

(a) Figure 4a shows a demonstration used in teaching simple harmonic motion. A sphere rotates in a horizontal plane on a turntable. A lamp produces a shadow of the sphere. This shadow moves with approximate simple harmonic motion on the vertical screen.

![Figure 4a]

(i) The turntable has a radius of 0.13 m and the teacher wishes the time taken for one cycle of the motion to be 2.2 s. The mass of the sphere is 0.050 kg. Calculate the magnitude of the horizontal force acting on the sphere.
(ii) State the direction in which the force acts. (3 marks)
(b) **Figure 4b** shows how the demonstration might be extended. A simple pendulum is mounted above the turntable so that the shadows of the sphere and the pendulum bob can be seen to move in a similar way and with the same period.

(i) Calculate the required length of the pendulum.

(ii) Calculate the maximum acceleration of the pendulum bob when its motion has an amplitude of 0.13 m.

(c) **Figure 5** is a graph of displacement against time for the pendulum.

\[ \text{displacement} \]
\[ \text{time} \]

**Figure 5**

Sketch, for the same interval, graphs of:

(i) acceleration against time for the bob, and

(ii) kinetic energy against time for the bob.

5

(a) Simple harmonic motion may be represented by the equation

\[ a = -(2\pi f)^2 x \]

(i) Explain the significance of the minus sign in this equation.

(ii) Copy **Figure 6** and sketch the corresponding \( v-t \) graph to show how the phase of velocity \( v \) relates to that of the acceleration \( a \).

(b) (i) A mass of 24 kg is attached to the end of a spring of spring constant 60 N m\(^{-1}\). The mass is displaced 0.035 m vertically from its equilibrium position and released. Show that the maximum kinetic energy of the mass is about 40 mJ.

(ii) When the mass on the spring is quite heavily damped its amplitude halves by the end of each complete cycle. Sketch a graph to show how the kinetic energy, \( E_k \), in mJ, of the mass on the spring varies with time, \( t/s \), over a single period. Start at time, \( t = 0 \), with your maximum kinetic energy.

You should include suitable values on each of your scales.
6 To celebrate the Millennium in the year 2000, a footbridge was constructed across the River Thames in London. After the bridge was opened to the public it was discovered that the structure could easily be set into oscillation when large numbers of people were walking across it.

(a) What name is given to this kind of physical phenomenon, when caused by a periodic driving force? 

(b) Under what condition would this phenomenon become particularly hazardous? Explain your answer.

(c) Suggest two measures which engineers might adopt in order to reduce the size of the oscillations of a bridge.

7 Figure 7 shows how the displacement of the bob of a simple pendulum varies with time.

![Graph of displacement vs. time](image)

(a) (i) Calculate the frequency of the oscillation.
(ii) State the magnitude of the amplitude of the oscillation.
(iii) State how the frequency and amplitude of a simple pendulum are affected by increased damping.

(b) Draw on a copy of Figure 7 the displacement–time graph for a pendulum that has the same period and amplitude but oscillates $90^\circ$ ($\pi/2$ radian) out of phase with the one shown.

(c) The pendulum bob has a mass of $8.0 \times 10^{-3}$ kg. Calculate:
(i) the maximum acceleration of the bob during the oscillation,
(ii) the total energy of the oscillations.

8 (a) A spring, which hangs from a fixed support, extends by 40 mm when a mass of 0.25 kg is suspended from it.
(i) Calculate the spring constant of the spring.
(ii) An additional mass of 0.44 kg is then placed on the spring and the system is set into vertical oscillation. Show that the oscillation frequency is 1.5 Hz.

(b) With both masses still in place, the spring is now suspended from a horizontal support rod that can be made to oscillate vertically, as shown in Figure 8, with amplitude 30 mm at several different frequencies.

![Diagram of spring and masses](image)

Figure 8

Describe fully, with reference to amplitude, frequency and phase, the motion of the masses suspended from the spring in each of the following cases.

(i) The support rod oscillates at a frequency of 0.2 Hz.
(ii) The support rod oscillates at a frequency of 1.5 Hz.
(iii) The support rod oscillates at a frequency of 10 Hz.